NUMERICAL STUDY OF MOIST BAROCLINIC WAVES

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1. INTRODUCTION

The direct effect of latent heat release on large scale flow is a warming of the ascending air where condensation occurs and a consequent enhancement of its buoyancy. This process increases the strength of the updraft, in the sense that positive maxima of vertical velocity become much larger than the descending values. The asymmetry in amplitude then becomes a spatial asymmetry, as the region of atmosphere occupied by ascending air becomes narrower than the descending area, in order to maintain continuity of mass. Finally, to complete the feedback cycle, vortex stretching by the strong and narrow updraft produces a more intense and smaller-scale vorticity perturbation than would be given in non saturated air. An approximate but effective way to include this process in a model of cyclone behaviour, and simple enough to allow partly analytic treatment, at least in the linear phase of growth, was introduced by Emanuel *et al.* (1987). If it is assumed that the ascending air is always saturated then the conservation of equivalent potential temperature gives a simple direct prortionality between the dry and moist derivatives (in time and horizontal directions) of potential temperature. The additional assumption that a slantwise convective adjustment occurs in the saturated air gives a value for the static stability of ascending air, which, in accord with the previous qualitative picture, is much smaller than the dry one.

The concept of slantwise convective adjustment provides that the lapse rate of saturated air, which we may imagine at some initial time as being moist-convectively unstable, is first reduced to neutrality by convective overturning, and then, being still unstable for moist symmetric perturbations, as it is common in a strong baroclinic environment, is further reduced to the point of slantwise neutrality. This occurrence has been observed (Emanuel, 1988) and numerically modeled (*e.g.* Kuo and Reed, 1988), but it is difficult to know if it is universal, and the time scales of slantwise overturning are not as fast as the convective time scales. Moreover the stability parameter for slantwise convection is the equivalent potential vorticity, which is a nearly conserved quantity, so that frictional or diabatic effects (other than the release of latent heat of condensation which is already taken care of in the definition itself) have to be invoked in its mixing.

With all the warnings mentioned we use the above assumptions to reveal in essential form the consequences of release of latent heat on baroclinic development. Two-dimensional studies were performed with analytic (Fantini, 1990) and numerical means (Fantini, 1993) before the full three-dimensional problem was tackled. We will now briefly describe the model, a longer presentation being considered for publication elsewhere (Fantini, 1997), and show preliminary results concerning the finite amplitude evolution of baroclinic cyclones in moist versus dry environment.

2. MODEL

In a previous work by the author (Fantini, 1995) a quasi-geostrophic model was used to obtain numerical evidence that moist baroclinic instability, when represented as explained in the Introduction, obeys the well known Squire theorem, *i.e.* for each unstable wave with given zonal length and a meridional structure there is a more unstable one with the same zonal (along stream) wavelength but infinite in the meridional (cross-stream) direction. In the course of that work, rather peculiar spatial structures were obtained for the moist normal modes. This result, and the realization that the strong updrafts induced in a saturated environment by the release of latent heat are coupled with a strong non-geostrophic circulation, caused some reservations on the validity of the quasi-geostrophic approximation in this context, although it was shown diagnostically that, at least in the linear regime, the non-geostrophic wind was mostly in the divergent rather than the rotational component, and therefore the solutions obtained were consistent. Having, however, realized that the proper dynamical setting for this problem is less balanced than the quasi-geostrophic one, and with the intent of also extending the results of Emanuel *et al.* (1987), and the subsequent ones by Joly and Thorpe (1989), which were semi-geostrophic, we proceeded with a primitive-equation formulation of the model. The equations being integrated are:

$$u_t+(\bar{u}+u)u_x+vu_y+w(\bar{u}_z+u_z)-f_o v = -c_p\theta_o\pi_x+\nu\nabla_H^2 u$$

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$$v_t + (\bar{u}+u)v_x + vv_y + wv_z + f_o u = -c_p\theta_o\pi_y + \nu\nabla_H^2 v$$

$$\theta_t + (\bar{u}+u)\theta_x + v(\bar{\theta}_y + \theta_y) + w(\frac{\theta_o}{g}r^* + \theta_z^*) = \nu\nabla_H^2\theta$$

$$u_x + v_y + w_z = 0$$

$$\frac{g}{\theta_o}\theta = c_p\theta_o\pi_z$$

The symbols have their usual meteorological meaning. We are using the formulation $-c_p\theta\nabla\pi$ for the pressure gradient force, where $\pi = (p/p_o)^{R/c_p}$, and the Boussinesq approximation is made by replacing θ by θ_o. The model is applied in an Eady-like setup, with constant vertical shear $\bar{u}_z$ and no meridional shear ($\bar{u}_y = 0$). The buoyancy term in the thermodynamic equation includes the representation of latent heat release discussed in the Introduction. Practically this amounts to using a piecewise constant static stability defined as

$$r^* = N^2 \equiv \frac{g}{\theta_o}\bar{\theta}_z \quad \text{where} \quad w < 0$$

$$r^* = N^2 \cdot r \quad \text{where} \quad w > 0$$

and $r << 1$ is computed by means of the slantwise convective adjustment hypothesis to be $r = \bar{u}_z^2/N^2 \equiv 1/Ri$

The non-linear term $w\theta_z^*$ presents a new problem with respect to the quasi or semi-geostrophic formulations which do not contain it. The problem is that, while the slantwise convective adjustment can realistically be assumed to have acted on the average state of the atmosphere, it is presumably too slow to neutralize the perturbed profile during the growth of the perturbation. Therefore in this model we choose to use the conservation of equivalent potential temperature in saturated air, and define

$$\theta_z^* = \theta_z \quad \text{where} \quad w < 0$$

$$\theta_z^* = \theta_{ez} \quad \text{where} \quad w > 0$$

The latter term can be completely calculated from

$$\theta_e = \theta + \frac{L}{c_p\bar{\pi}}q_v^s$$

since the saturation mixing ratio q_v^s is only a function of the two thermodynamic variables θ and π which are known by the model.

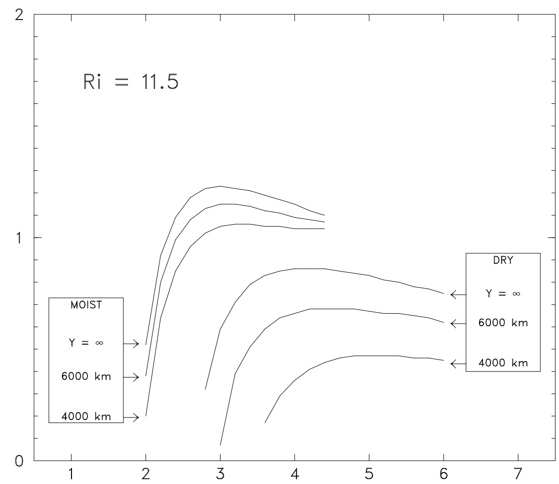

Figure 1. Growth rate in units of $10^{-5}\mathrm{s}^{-1}$ versus zonal wavelength in thousands of kilometers, in an environment characterized by Richardson number 11.5, for three different meridional wavelengths, in the dry and moist cases, as labeled. The buoyancy enhancement factor for this environment is $r = 1/Ri = .087$

3. RESULTS

We show in Fig.1 the dimensional growth rate versus zonal length for three different meridional structures in the dry and moist case. The most unstable is clearly the two-dimensional wave ($Y = \infty$), and the growth rate is reduced as the meridional wavelength Y is reduced. The variability with meridional wavelength, both in growth rate and length of the most unstable mode, is seen to be much reduced in the saturated environment. Growth rate increase from dry to moist case goes from about 50 percent for the 2D wave, to about 150 percent for the waves with 4,000 km meridional wavelength. The zonal length of the most unstable mode is also reduced in larger measure when modes with a meridional structure are considered.

Fig.2 shows the surface pressure of a dry and a moist normal mode in the reference environment of $Ri = 11.5$. Wavelengths are 4000 km zonal and 6000 meridional. The dry mode is taken as initial condition for the run that generates the moist one after a time of adjustment as can be seen from the growth rates in Fig.3. An high-low asymmetry is evident from the figure, while other structural changes are not as large for this wavelength as they would be for either longer or shorter waves, but a tendency for the low pressure area to extend in the meridional direction can be seen. This behaviour

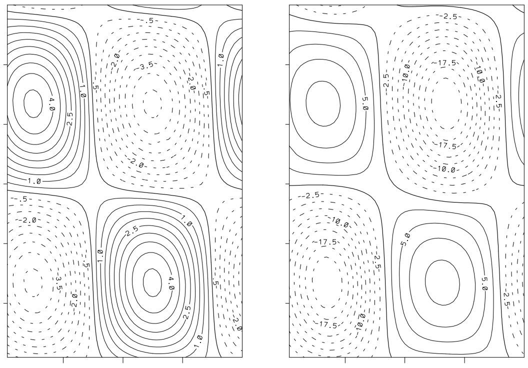

Figure 2. Left: surface pressure for the dry normal mode of 4000 km zonal wavelength and 6000 km meridional wavelength. Initial condition for all the experiments shown. Right: same for moist mode obtained from dry one after 100 hours of moist integration.

is enhanced for the longer modes (not shown), to the point that, for waves longer than twice the most unstable one, the low pressure areas join up in the meridional direction giving way to the two-dimensional wave of half zonal length. For shorter waves, instead, the low pressure areas are elongated in the zonal direction (not shown), but only near the surface, and the mid-level fields still exhibit the well defined wavelength imposed initially.

Fig.3 shows the evolution of growth rate during the experiments started at the same initial condition. The first 100 hours are linear runs in the dry and moist environments. The dry run does not exhibit any change in growth rate because the initial condition is already the dry normal mode. In the saturated environment the solution goes through an adjustment phase of little more than 70 hours, after which time it settles down in the moist normal mode. Both the dry and the moist mode after 100 hours of this run are those shown in Fig.2. At 100 hours, and after a reduction in amplitude to make the subsequent evolution directly comparable, the non-linear terms are switched on. The phase of faster growth of the non-linear moist mode is rather characteristic, and was earlier noted by Whitaker and Davis (1994).

Fig.4 shows the surface pressure fields at comparable amplitudes (reached by the dry run at a much later time). The zero line is rather similar in shape but the minimum pressure in the moist case is less centered than the dry one, and the gradients to the north of the minimum, and consequently the geostrophic wind, stronger. After the time shown the dry perturbation deepens further (not shown) but assumes an even more symmetric, less realistic, shape.

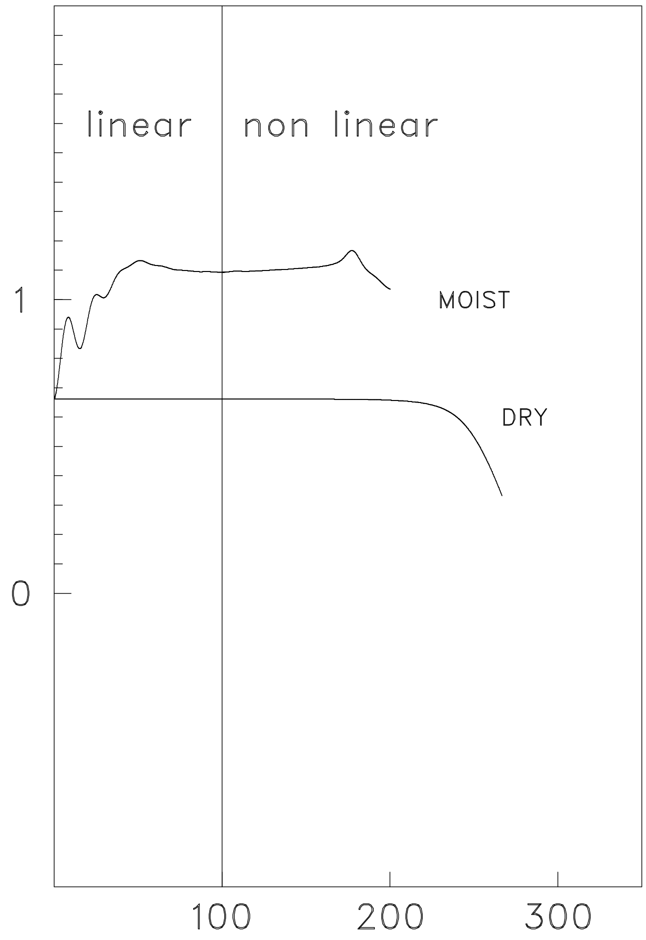

Figure 3. Growth rate in units of $10^{-5}s^{-1}$ versus time in hours for a dry (lower curve) and a moist experiment (upper curve) started from the dry normal mode shown in Fig.2. The first 100 simulated hours are linear and the amplitudes are normalized at 100 hours so that the following non-linear integration should be comparable.

4. CONCLUSION

We have shown preliminary results for the evolution of baroclinic cyclones in a saturated environment, obtained numerically with a primitive equation model and the parameterization of Emanuel *et al.* (1987) for the effects of latent heat release on the buoyancy term of the thermodynamic equation. The results show a faster growth in the saturated environment, but for most of the evolution the spatial structure of the surface pressure field for a moist cyclone does not differ in an essential way from the dry one, except that the dry simulation takes much longer to attain the same stage of devel-

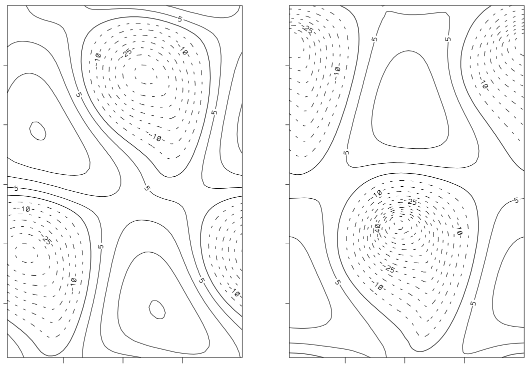

Figure 4. Left: surface pressure for the dry run at 250 hours, *i.e.* 150 hours into the non-linear integration. Right: surface pressure for moist run at 200 hours.

opment. When the two cases appear to diverge also in shape, as mentioned above for the latest stages of the dry experiment, a comparison is not possible because the moist experiment blows up, presumably due to frontal collapse. It must be mentioned that some artificial dissipation in the form of a horizontal laplacian of the prognostic fields is applied with the same intensity in the two runs and this is likely to be the cause of the unrealistic latest stages of the dry cyclone, when dissipation overcomes the now minimal growth and prevents further development. More careful comparison of the mature and decaying stages of baroclinic life-cycles are in progress.

The comparison was here effected at the fixed wavelength of 4000 km, this being (see Fig.1) in the rather restricted range where both dry and moist modes are significantly unstable. A situation initialized with a white signal, rather than a specified wavelength, would give rise to the most unstable of the modes represented, and therefore differences in the horizontal size of the cyclone in the two environments would appear. Moreover, even for the rather similar solutions presented here (Fig.2), other fields exhibit more differences, the most notable of them being the vertical velocity, which behaves as mentioned in the Introduction.

Finally, it should also be mentioned that while we always use the term 'linear' to indicate the 'small amplitude' approximation, the release of latent heat is intrinsically non-linear, and this fact is represented in our equations in the dependence of the buoyancy on the sign of w, so that the superposition principle is invalid and our use of the term 'normal mode' only refers to the exponential growth of a separate spatial structure.

5. ACKNOWLEDGMENTS

This work was supported by EU contract EV5V-CT94-0442 'ANOMALIA'.

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