Discrete representation and resampling in limb-sounding measurements

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The processes of discretization, interpolation, and resampling are frequently used in data analysis. Here the formalism of functional spaces is used as a framework for the description and characterization of both the measurement operation and these subsequent processes. The tools provided by this formalism are applied to the problem of resampling of atmospheric volume mixing ratio vertical profiles obtained with limb-sounding measurements. In particular, a resampling method that uses the conservation of the vertical column as a constraint is presented and compared with other methods. The effects of the resampling process in terms of error propagation and loss of vertical resolution are also evaluated.

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1. Introduction

Both measurement and numerical modeling involve the discretization of quantities that are physically described by continuous functions. In numerical modeling, the discretization process can be considered not to provide a constraint because by means of interpolation it is always possible to reconstruct the continuous function. Even if the selected interpolation rule does not always ensure a reconstruction of good quality, one can in principle achieve adequate quality by starting from a sufficiently fine sampling grid.

In measurements, the discretization process is constrained by the sampling grid of the measurements. This is the case, for instance, of the retrieval of atmospheric constituents' vertical profiles from radiometric limb-sounding measurements. The quantities to be retrieved are continuous functions, but the available measurements are finite in number and furthermore are often not fully independent among themselves. In this case the discrete representation of the profile retrieved from the measurements may be characterized by a sampling grid that is rather coarse for proper representation of the function.

In the subsequent data utilization, if the user's sampling grid is different from the measurement sampling grid, a resampling is necessary, and this process is often an irreversible operation that causes loss of information.

Discretization, interpolation, and resampling certainly represent an old problem but one that is still a crucial issue in many fields of applied science: for image manipulation and reconstruction, in medical applications, in processing of satellite data, and for data analysis. It is a key issue in meteorological forecast and data assimilation, for which it is necessary to start from a model of the atmosphere defined on an appropriate altitude and latitude grid for which available measurements are generally sparse and irregular. All these applications involve the handling of large data sets that are often two dimensional, and, depending on the properties of the data, many different interpolation rules can be used.

These rules include the use of analytical convolution kernels of different degrees of complexity (nearest neighbor, linear, cubic, nth-order polynomial, windowed sinc) or statistical data processing (e.g., Kriging optimum interpolation). In these applications the process of resampling is usually considered to coincide with the process of interpolation combined with the disposal of the original points, without evaluation of the possibility of optimizing the resampling process as a process to be performed independently of the interpolation.

In this paper we consider resampling in the case of
geophysical data for the one-dimensional case, focusing on the problem of the loss of information that occurs in each resampling step. A mathematical analysis of the measurement process and of the subsequent transformations is made in Section 2. With the formalism of functional spaces the measurement process can be viewed as the determination of the unknown function in the subspace of the measurement functions. Some data-retrieval processes involve the representation of the measured function in a new space, the so-called retrieval space, and hence require a transformation from one space to another. One can also use the formalism of the transformation from a functional space to another to describe the resampling of a function and provide a rigorous description of this operation.

The process of resampling is analyzed in the case of the near real-time processor of the Michelson Interferometer for Passive Atmospheric Sounding (MIPAS) instrument that will fly on the Environmental Satellite (ENVISAT). In Section 3 we describe and compare some resampling methods developed for handling the atmospheric vertical profiles retrieved by the analysis of the atmospheric limb-emission spectra detected by the MIPAS instrument. In particular, a method is presented that uses the conservation of the vertical column as a constraint for the resampling process.

The effects of resampling in terms of error propagation and loss of vertical resolution are evaluated in Section 4.

2. The Mathematics

A. Discrete Representation

The discretization of a function is usually considered to coincide with the operation of tabulation, that is, with the sampling of the function at a finite number of values of the independent variable. In principle, the spacing of the sampling grid can be made as fine as necessary such that all the features of the function can be fully represented.

In the measurement process we try to reproduce this discretization by making measurements of a function at values of the variables that are spaced as closely as possible and by limiting the width of the interval in which the instrument integrates, such that nearly punctual values of the unknown function are determined. This is typically what happens in spectroscopy when the measurement of a spectrum requires the use of close sampling points and a sufficiently narrow spectral resolution. However, the measurement process is in general more elaborate than the tabulation process.

The measurement of a function not only is the measurement of the value of the function in small intervals of integration but more generally is the measurement of the total integral of the unknown function filtered by an instrument-specific function that we can call the measurement function. The result of this operation is known in mathematics as the inner product of two functions. Using vector notation, we can write this operation as

\[ m_i = \langle g_i, f \rangle, \]

where \( f \) is the function to be measured and \( g_i \) is the measurement function that corresponds to the \( i \)th measurement \( m_i \).

The measurement process can be described as the determination of the components of unknown function \( f \) with respect to measurement functions \( g_i \). The set of the measurement functions defines a functional space \( \{g_i\} \), which is identified as the measurement space. If \( \gamma_i \) is a complete base of the space \( \{g_i\} \), any function \( f' \) that belongs to that space can be expressed as

\[ f' = \Gamma c, \]

where \( \Gamma \) is the matrix whose columns are given by the base functions \( \gamma_j \) and \( c \) is the vector of the coordinates of \( f' \) with respect to the base functions. Boldface type is used to indicate functions, vectors, and matrices. The matrix notation is used also to indicate a set of ordered functions. The matrix whose columns contain the functions of a given base is identified by a capital letter that corresponds to the lowercase letter of these functions.

Base \( \gamma_j \) is in principle a rather general set of functions, but in the following discussion we assume that either it coincides with the set of measurement functions \( g_i \) (when these are linearly independent) or it is otherwise a suitable subset of them. If \( f' \) is the representation of \( f \) in the space \( \{g_i\} \), from Eqs. (1) and (2) it follows that

\[ m = G^T \Gamma c. \]

The least-squares solution of Eq. (3) is given, for measurements with independent and constant error, by

\[ c = (G^T \Gamma)^{-1} m, \]

where \( ^{-1} \) indicates the generalized inverse of the matrix. From Eqs. (2) and (4) we obtain

\[ f' = \Gamma (G^T \Gamma)^{-1} m. \]

Equation (5) provides the most general representation of the measurement of a function.

An example of the possible use of this general representation is provided in Ref. 12, which describes its use for a comparison of the indirect measurements of Fourier-transform spectroscopy and the direct measurements of grating spectroscopy.

In the case of direct measurements the complexity of Eq. (5) is lost because the measurement functions \( g_i \) are made similar to contiguous boxcar functions. As a consequence the functions \( \gamma_j \) coincide with the functions \( g_j \), the matrix \( (G^T \Gamma)^{-1} \) is the unit matrix, and Eq. (5) is reduced to the expression of a histogram of the functions \( g_j \) with individual amplitude \( m_j \).

In the case of indirect measurements, Eq. (5) is ei-
ther the expression that provides the transformation into direct measurements (e.g., Fourier-transform spectroscopy and Hadamard spectroscopy) or the expression of the linear inversion process (e.g., retrieval of limb-sounding measurements).

At the beginning of this section we noted that the discretization of a function is usually considered to coincide with the operation of tabulation. However, because the measurement process is one of the practical cases in which the discretization of a function is performed, and because Eq. (5) is a general representation of the measurement process, we conclude that tabulation is only a particular case of discretization, and a more general description of the discretization process is provided by Eq. (5).

Even if the solution provided by Eq. (5) is represented in a functional space with finite dimension, it provides a continuous representation of the unknown function as long as the measurement functions are known as continuous functions. In retrieval problems this knowledge is often not practically available; furthermore, the modeling of the measurement process may require the representation of $f$ in a base different from the measurement function base. In this case a change of representation must be considered part of the retrieval process. This further generalization is considered in Subsection 2.B.

B. Change of Representation

The change of representation implies that, even if the measurement is made in the space of the measurement functions $\{g_j\}$, its representation is sought in a space of different functions $\{h_j\}$, to which we refer in the following discussion as retrieval functions, where a new estimate $f''$ of function $f$ is defined as equal to

$$f'' = H d.$$

We can determine the coefficients $d$ of this new representation either by imposing the condition that $f'$ and $f''$ have the same components in the space of retrieval functions $\{h_j\}$ or by imposing the condition that $f'$ and $f''$ have the same components in the space of measurement functions $\{g_j\}$,

Condition (a) $\{h_j, f'\} = \{h_j, f''\}$,  
Condition (b) $\{g_j, f'\} = \{g_j, f''\}$,  

respectively, for all vectors $h_j$ and $g_j$ of the two spaces.

If both $g_j$ and $h_j$ are complete bases, the solutions that are obtained from conditions (7) and (8) are, respectively,

$$f''_a = H (H^T H)^a H^T G (G^T G)^a m,$$  
$$f''_b = H (G^T H)^b m.$$

If the two sets of functions $h_j$ and $g_j$ identify the same space, the two conditions coincide, and the two new estimates are equal to the old one:

$$f''_a = f''_b = f'.$$  

However, in general the new representation is different from the original one ($f'' \neq f'$), and two different solutions are possible ($f''_a \neq f''_b$).

The implications of the two solutions are illustrated by the example provided in Fig. 1 for the case of vectors in a plane. A new representation along the versor $h$ of the vector $F'$, which is originally determined along the versor $g$, can be either the vector $F''_a$ (constant component in the retrieval space) or the vector $F''_b$ (constant component in the measurement space). Both transformations involve a significant modification that depends on the difference between the measurement space and the retrieval space (in this case of Fig. 1, on how large the angle is between the two versors). From Fig. 1 it is also evident that transformation of type a produces a reduction in the amplitude of the components of vector $F'$ and hence in general a loss of detail, whereas transformation of type b produces an amplification of the amplitudes and hence of the errors. In the limit that vectors $h$ and $g$ are normal among themselves, $F''_a$ goes to infinity, whereas $F''_b$ is equal to 0. We shall refer to the transformation of type a as the classical transformation and to the transformation of type b as the exuberant transformation.

The retrieval of the function in a space different from that of the measurement functions may be motivated by a wish to determine $f$ in a base of retrieval functions that have the property of being as similar as possible to boxcar functions, so the coordinates in this space provide a histogram of the measured function. However, often the different representation is also motivated by the computing difficulty of determining the measurement functions with all the desirable details. This is the case for remote-sensing measurements in which the calculation with a sufficiently fine discretization of $(G^T G)^a$ in Eq. (5) may require a long computing time. In this case the calculation of $(G^T G)^b$ in Eq. (9) also requires a long computing time, and only the exuberant solution of type b can be obtained. It can be noted that Eq. (10)

![Figure 1](image-url)
is the formula that is commonly used in inverse problems, where the product $G^T H$ is the discrete Jacobian that one can directly calculate without passing through the determination of its terms.

We conclude also that the discrete representation of indirect measurements is often made with some boxcar-type functions. When a boxcar-type function can be found in the measurement space (e.g., in Fourier-transform spectroscopy the sinc functions with which the spectra are represented belong to the same space of the cosine measurement functions) the solution is provided by Eq. (5). If the boxcar-type functions can be found only in another functional space $\{h_j\}$, the solution can be provided by either Eq. (9) or Eq. (10). In the case of limb-scanning retrievals, computational constraints prevent the use of transformation (9), and transformation (10) is always used. The deformation introduced in general by the change of representation and in particular by the exuberant character of transformation (10) adds to the indetermination of retrieval problems and contributes to the instabilities that are often observed in the retrieved quantities.

C. Interpolation

In light of the above characterization of the discrete measurement of a function, here we discuss the operation of interpolation.

Interpolation is the process by which, from a given discrete representation of a function, the values of the function are determined at new points of the independent variable. Interpolation involves the determination of the curve that can be drawn through the discrete values of the function. The new interpolated values can be determined on this curve. Therefore the problem of interpolation usually coincides with the so-called drawing problem of determining the best interpolating curve.

However, the results obtained in Subsection 2.B indicate that the most rigorous process for the reconstruction of the continuous distribution, and therefore for interpolation, is provided by Eq. (5) and Eqs. (9) and (10), which define how the continuous representation can be obtained from a linear combination of either the measurement functions or the retrieval functions. In some cases the two processes of drawing interpolation and mathematical reconstruction coincide, but usually they provide quite different results.

D. Resampling

Another process that is often needed for the exploitation of a measurement is a change of the sampling grid. For instance, such is the case when in spectroscopy we compare two spectra measured on different frequency grids and for this purpose one of the two spectra is resampled at the grid of the second spectrum.

In Subsection 2.C we saw that either of two approaches can be used for the interpolation problem. Similarly, either of two approaches can be used for the resampling problem. The resampling process can be heuristically implemented with an interpolation for determination of the values at the new grid followed by disposal of the values at the original grid (heuristic resampling). Alternatively, the resampling process can be implemented with the more general operation of change of representation that we saw in Subsection 2.B.

In Subsection 2.B we discussed the change of representation for the transformation from measurement functions to retrieval functions. Similarly, a further transformation from retrieval functions to user functions can be performed. Also, this change of representation is always possible.

When the retrieval functions and the user functions identify the same space, only one transformation is possible and is made without loss of information. This is the case, for instance, for a shift in the frequency grid of a spectrum obtained with Fourier-transform spectroscopy, because the sinc functions define a space of functions to which shifted sinc functions also belong. When the two bases do not belong to the same space, a choice must be made between the two different transformations (either classical or exuberant transformation). A practical example is discussed next.

3. Practical Case of Geophysical Data

The MIPAS instrument is a high-resolution Fourier-transform spectrometer developed by the European Space Agency (ESA) that will detect atmospheric limb emission in the middle-infrared at tangent altitudes from 8 to 53 km, in the nominal case. The spectra measured in each limb-scanning sequence are routinely processed to yield the temperature and pressure profiles as well as the volume mixing ratio (VMR) profiles of numerous atmospheric trace species.

In the ESA's Level 2 near-real-time-processor these profiles are retrieved on an altitude grid defined by the tangent altitudes of the measured limb spectra. This choice is dictated by the fact that in correspondence with these altitudes the VMR is measured with the best accuracy and resolution.

In the retrieval process the observed spectra are fitted with calculated spectra obtained with a radiative transfer model through a nonuniform atmosphere. In the radiative transfer model the retrieved values of the VMR are linearly interpolated for the determination of the atmospheric composition at intermediate altitudes. This choice corresponds to the use of a retrieval space made from triangular functions. The functions drawn as solid lines in Fig. 2 provide examples of the functions that belong to this space.

It is expected, however, that users of MIPAS measurements may need to represent the VMR profiles on a different (user-defined) sampling grid. It is important, therefore, to understand what the effect is of different resampling methods and to choose the most efficient one.
A. Standard Resampling Methods

Figure 2 depicts, together with the functions that form the base of the retrieval space, the base of a possible user space and provides an example of possible pessimistic relative locations of the functions in the two spaces.

Figure 3 shows the results obtained from three different techniques of resampling for the case of a typical nitric acid (HNO₃) VMR profile. The original profile, shown by the solid curve, shows some oscillations at high altitude, which are due to retrieval error, and a pronounced peak at intermediate altitudes whose amplitude and location should be accurately measured. The result of the heuristic resampling is represented by the circles, the result of the classical transformation obtained from condition (7) is represented by the squares, and the result of the exuberant transformation obtained from condition (8) is represented by the triangles. At first sight it seems that the three techniques are rather similar and give comparable results, but some differences can be noted at those altitudes where the original profile has some oscillations and where it peaks (that is, whenever the second derivative of the original profile is different from zero). In particular, the exuberant transformation shows a tendency to amplify the existing oscillations.

To make the features of the different transformations more evident, we performed a test in which each transformation process was applied consecutively three times. The grids used in each transformation were randomly chosen. The results are shown in Fig. 4. In this case the exuberant transformation introduces oscillations that extend out of the scale of the plot, and the corresponding curve is no longer plotted. The heuristic resampling, because of its tendency to smooth the distribution, successfully removes the oscillation at high altitudes but also leads to an undesirable underestimate of the values at the peak of the distribution. The classical transformation, instead, maintains a realistic information about the shape and the values of the distribution and proves to be the best resampling method of the three considered.

From Fig. 2 it is evident that both the retrieval space and the user space include a function with a constant value equal to the summation of the functions of each base. This implies that the component of the profile with respect to a constant function, that is, the integral of the VMR profile, is conserved in the case of both the classical and the exuberant transformations. Numerical tests confirm this result, even for the profile obtained after three exuberant transformations. This is in general a desirable mathematical property but does not correspond to the physical properties of our measurements. The measured quantity of the MIPAS instrument is the atmospheric radiance emitted by a column of gas in the line of sight. Therefore the quantity that should be conserved is the integral of gas concentration, equal to the vertical column, rather than the integral of a mixing ratio that does not correspond to an observed quantity.

In the light of this consideration, we shall now consider an alternative transformation from retrieval space to user space.
B. Resampling with the Constraint of Constant Columns

Considering that the column of each gas between two altitudes is the quantity actually determined by the retrieval analysis, instead of conserving the components of the function in one of the two spaces [Eqs. (7) and (8)] we can perform resampling by imposing the conservation of the columns at the retrieval grid points.

The vertical column \( \text{Col} \) above a particular altitude \( z_1 \) is defined as

\[
\text{Col} = \text{const} \int_{z_1}^{z_{\text{lim}}} X_{\text{gas}}(z) \frac{P(z)}{T(z)} \, dz,
\]

where const is a constant, \( z_{\text{lim}} \) represents the upper boundary of the atmosphere, and \( P(z) \), \( T(z) \), and \( X_{\text{gas}}(z) \) are the pressure, the temperature, and the gas VMR's, respectively. In Eq. (12) the quantities \( X_{\text{gas}}(z) \), \( P(z) \), and \( T(z) \) are assumed to be continuous functions of the altitude \( z \).

The strategy adopted to perform the resampling with the constraint of column conservation consists in determining the values of VMR on the new grid such that the partial columns between points of the retrieval grid are invariant. It has to be noted that we assume that the number of grid points in the user-defined grid is not greater than the number of grid points in the retrieval grid. Because the dependence of the partial columns on the values of VMR at the user-defined grid levels is linear, it can be expressed by a matrix.

The procedure used to resample the VMR profile at the user-defined grid is the following one:

1. Calculate partial columns \( \mathbf{c} \) between points of the retrieval grid as a function of pressure, temperature, and VMR. The linear relation between the columns and the VMR-retrieved profile \( \mathbf{x} \) is expressed by matrix \( \mathbf{D} \):

\[
\mathbf{c} = \mathbf{Dx}.
\]

2. Resample pressure and temperature at the user-defined grid. To this purpose, an algorithm consistent with the results of this study should be used, but for simplicity in the following numerical examples heuristic interpolations (respectively logarithmic and linear with altitude) are used.

3. Compute the matrix \( \mathbf{E} \) that provides the relationship between the partial columns at the retrieval grid and the VMR values \( \mathbf{x}_r \) at the user-defined grid:

\[
\mathbf{c} = \mathbf{Ex}_r.
\]

4. Compute the VMR values at the user-defined grid, inverting the matrix \( \mathbf{E} \):

\[
\mathbf{x}_r = \mathbf{E}^{-1} \mathbf{c} = \mathbf{E}^{-1} \mathbf{Dx}.
\]

If matrix \( \mathbf{E} \) is a rectangular matrix (or in general if it cannot be inverted), the generalized inverse matrix can be used instead of the inverse.

If \( \mathbf{L} \) is the base of the user-defined space in which \( \mathbf{x}_r \) is determined and \( \mathbf{H} \) is the base of the retrieval space in which \( \mathbf{x} \) is determined, the relationship between continuous resampled profile \( \mathbf{f}_r \) and continuous retrieved profile \( \mathbf{f} \) is given by

\[
\mathbf{f}_r = \mathbf{Lx}_r = \mathbf{LE}^\dagger \mathbf{DH} \mathbf{x} = \mathbf{LE}^\dagger \mathbf{DH} \mathbf{f}.
\]

The profile obtained by use of the transformation that maintains the columns invariant is similar to that obtained with the classical transformation, and to appreciate the similarities and the differences we show in Fig. 5 the result obtained with the two techniques when they are applied repetitively three times.

The MIPAS tool for resampling the retrieved profiles on a user-defined altitude grid will use the transformation with the constraint of constant column for its more rigorous physical meaning; however, for most practical purposes the classical transformation also appears to provide good results. The use of the heuristic transformation and that of the exuberant transformation, as shown in Section 3, are not recommended.

4. Error Propagation and Vertical Resolution

When we perform a transformation, as in the case of interpolation and resampling, we have to consider how this operation affects the trade-off between vertical resolution and error of the retrieved profile.

As described in Ref. 15, the loss in vertical resolution of a transformation can be measured by use of the modulation transfer function (MTF), which is equal to the Fourier transform of the rows of the transformation matrix. From the combination of Eqs. (1) and (10), we see that the retrieval of profile \( \mathbf{f}_r \) can be described as a transformation \( \mathbf{T} \) applied to the real profile \( \mathbf{f} \):

\[
\mathbf{f}_r = \mathbf{H(G}^\dagger \mathbf{H})^\dagger \mathbf{G}^\dagger \mathbf{Tf} = \mathbf{Tf}.
\]

Each row of transformation matrix \( \mathbf{T} \) contains information on how much the retrieved solution at a given altitude depends on the actual values of the profile at
the other altitudes. Therefore the deviation of this row from the Dirac delta distribution gives a measure of the vertical resolution that can be achieved at the corresponding altitude. The vertical resolution can be conventionally defined as the reciprocal of twice the frequency at which the value of the MTF drops to half of its starting value. The resampled function is obtained by application of a further transformation \( P \) to the retrieved function \( f_r \):

\[
f_u = Pf_r = PTF_r.
\]  

(18)

The MTF of transformation \( P \) provides the vertical resolution of the whole process, and the MTF of \( P \) provides the vertical resolution of the resampling process. Therefore, even if the final performance of the transformation \( P \) depends on the interactions of \( P \) with transformation \( T \), the study of the \( P \) transformation provides a good indication of the properties of the resampling process.

In the case of the classical transformation [Eq. (9)], because \( L \) is a base and therefore is made from linearly independent vectors, \( P \) is given by

\[
P = L(L^T L)^{-1} L^T = LL'.
\]  

(19)

In the case of the transformation that maintains the column invariant [Eq. (16)], \( P \) is given by

\[
P = L'E'DH'.
\]  

(20)

In Figure 6 the vertical resolutions of the two resampling transformations are compared as a function of altitude. The minima of the curves occur in correspondence to the user-defined grid levels; the maxima correspond to the intermediate altitudes, which, as they are calculated as a combination of other values, involve an averaging process and are affected by a further broadening.

The vertical resolution at the resampled points is similar for the two transformations and greater than 4 km. As it is evident from Eq. (19), the loss in vertical resolution that is due to resampling in the case of the classical transformation is independent of the old grid \( (H) \) but depends on the relative spacing of the new grid \( (L) \). In contrast, for the transformation that keeps the columns invariant, the loss in vertical resolution shows a small variation with altitude and depends on the relative position between the two grids \( (H \) and \( L) \).

Inasmuch as the vertical resolution of the retrieved points is of the order of the spacing of the retrieval grid (~3 km), the loss of vertical resolution caused by resampling is a conspicuous and dominant effect.

For the assessment of error propagation, discrete quantities must be considered. The variance covariance matrix of the resampled profile \( x_u \) is given by

\[
V_u = KV_r K^T,
\]  

(21)

where \( K \) is the transformation matrix from the discrete retrieved profile \( x_r \) to the discrete resampled profile \( x_u \) and \( V_r \) is the variance covariance matrix of the retrieved profile. Assuming in a first approximation that \( V_r \) is diagonal, we can estimate the change in the measurement error introduced by the resampling process from the values of the diagonal of the matrix:

\[
C = KK^T.
\]  

(22)

For the classical transformation, considering again that \( L \) is a base and therefore is made from linearly independent vectors, \( K \) is given by

\[
K = (L^T L)^{-1} L^T H = L' H,
\]  

(23)

whereas for the transformation that maintains columns invariant \( K \) is given by [see Eq. (15)]

\[
K = E'D.
\]  

(24)

Using these two transformation matrices, in Fig. 7 we compare the estimated changes of measurement error introduced by the two resampling processes. On average, a small reduction in the measurement error is
error is observed in both cases. The calculations of vertical resolution loss and of error propagation show that the two resampling methods give very similar results and underline that the resampling process is characterized by a small reduction in measurement noise but a significant loss of vertical resolution.

5. Conclusions

The formalism of functional spaces provides a useful tool for describing the process of measurement of a function and for handling the information in subsequent transformations. The measurement process is the determination of the unknown function in the subspace of the measurement functions. Some data-retrieval processes such as those used for the analysis of limb-sounding atmospheric observations involve the representation of the measured function in a new space (retrieval space). The change of space can be made with either a classical transformation, which determines the component in the new space of the function measured in the old space, or an exuberant transformation, which determines, in the new space, the function that in the old space has the measured component. The latter transformation introduces undesirable amplifications of some components of the function representation. The transformations from measurement space to retrieval space that are performed in limb-sounding retrieval processes, always involve exuberant transformations, which explain some of the instabilities observed in retrieved profiles. A transformation from one space to another is also involved in the resampling of a function. A comparison was made of various resampling methods. The classical transformation is the recommended one because it preserves the shape of the original function, whereas the exuberant transformation may induce large oscillations. The heuristic transformation, based on linear interpolation, causes loss of detail and does not conserve the integral of the function.

In the special case of vertical VMR profiles the resampling process can be usefully constrained by the conservation of the vertical column. The resampling method based on this technique provides results that are quite similar, in terms of shape reconstruction, vertical resolution loss, and error propagation, to those obtained with the classical transformation but is a sound alternative to the latter because it is based on more-rigorous physical grounds. The use of the resampling process should, however, be limited as much as possible, because it changes the trade-off between retrieval error and vertical resolution, leading to some small improvement of the former and to significant deterioration of the latter.

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