Theoretical and practical consideration of the construction of a zero-geometric-loss multiple-pass cell based on the use of monolithic multiple-face retroreflectors

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The theory of the multiple-pass cell based on the use of retroreflectors is presented. As a result of this study, it is shown that it is possible to construct an enhanced White cell with zero geometric loss. Starting from theoretical considerations of the design of a new monolithic multiple-face retroreflector, a multiple-pass cell is proposed. Ray-tracing simulations indicate that this cell is easy to align and has zero geometric loss over a very long optical path. © 2001 Optical Society of America


1. Introduction

The three-mirror multiple-pass cell, known as the “White” cell, has been used extensively in spectroscopy since the first paper by White1 was published in 1942. Its applications include spectroscopic measurements that require long absorption paths to measure the presence of trace gases and especially those with small absorption cross sections.

The White cell consists of three concave mirrors with equal radii of curvature $R$: one field mirror and two adjacent objective mirrors opposed to the field mirror at a distance equal to $R$. The center of curvature of the field mirror lies halfway between the two objective-mirror surfaces, and the centers of curvature of the objective mirrors lie on the field-mirror surface. The optical aperture of the White cell is fixed by the focusing of the input beam by the first objective mirror; hence, the White cell is suitable for use with large-aperture sources. The main purpose of the White cell is to reproduce the image of the source after each double pass of the radiation between the field and objective mirrors. The reimagining optical design keeps all rays collected in the first pass within the mirrors until the exit pass, and therefore the White cell has zero geometric optical loss. Beam radiation is only reduced by interaction with the gas sample between the mirrors and by absorption at the mirror surfaces.

Several variations of the White design have been proposed by authors to improve the performance of the system.2–6 In 1976 White7 himself introduced two retroreflectors to his original design to reduce the influence of misadjustment of the objective mirrors on the exit-image position. This improvement allowed an increase in the number of passes of a large-aperture beam, reducing the negative effects of mechanical vibrations, thermal expansion, and sample inhomogeneity.

Ritz et al.8 further developed the design with three quartz prisms. As shown by White,7 Ritz, and Doussin et al.,9 the enhanced prism configuration of the cell achieves two goals: It compensates for the White cell misalignment and allows more passes by using mirrors of the same size.

When the light coming from an objective mirror is intercepted by a prism, it is shifted horizontally or vertically, depending on the prism configuration, and is then returned to the objective mirror itself. If one of the objective mirrors is misaligned slightly in the vertical or the horizontal plane, each image focused on the field mirror is offset from its original position. This offset grows progressively at every successive reflection at this objective mirror.

Once the light has crossed a prism, it is reflected back with the same angular direction in the plane perpendicular to the prism shift, and the offset on the objective mirror is reversed. Each successive travel
back and forth among the spherical mirrors progressively reduces the misalignment in the plane perpendicular to the prism shift, whereas the misalignment in the plane parallel to the prism shift continues to grow.

When two prisms mutually rotated at 90 degrees are combined, the misalignment is cancelled completely. Thus the alignment of the objective mirrors needs only to be accurate enough to make the images fall on both the field mirror and the prisms.

The introduction of the retroreflectors in the original White design has been done by White, by Ritz, and by Doussin by an approximate optical design. Each researcher has neglected the effect induced in the beam divergence of the White cell by reflections on the planar elements. Furthermore, Ritz and Doussin have used average optical constraints to calculate the aperture angles of the prisms.

As we show in the following chapters, the approximations of these researchers exert a negative effect on the light throughput by increasing the geometrical optical loss. In particular, when a 2-m White cell of f/40 optical aperture is designed, the solution proposed by Ritz leads to an optical design with zero light transmission.

This paper describes the exact solution of the enhanced White cell with retroreflectors. From a mathematical point of view, we introduce the relationships to describe correctly the retroreflector properties. Furthermore, we propose the introduction of new optical elements that allow the present White cell to operate at zero geometrical optical loss without increasing the complexity of the system.

2. Optical System Design and Theory

We start with Ritz’s multiple-pass cell, shown in Fig. 1. The three quartz prisms, used in a total-reflecting configuration, are mounted near the field mirror.

In this optical design the entrance, the centers of curvature of the two objective mirrors, and the exit are not aligned. The resulting pattern of the light spots on the field surface for the 144-pass configuration is shown in Fig. 2.

As shown by Ritz and Doussin the misalignment-free condition is reached at a focus labeled as 40 (Fig. 2). The light returned from prism F is parallel to the incoming beam, and that prism essentially is used to duplicate the number of passes.

A crucial point of the enhanced White cell with prisms is that the aperture angle \( \alpha \) (i.e., the angle of the prism opposite to its hypotenuse) and the alignment of each prism should be set to allow the incident beam to return to the objective mirror, overlapping exactly its source point.

To fully describe the optics we consider a coordinate system with the origin at the center of the field mirror surface: the \( x \) axis along the White cell optical axis, the \( y \) axis parallel to the axis along the centers of the two objective mirrors (horizontal), and the \( z \)-axis (vertical).

Once the size of the prism-field surface (i.e., the prism surface opposite its aperture angle) and the prism position in the \( y-z \) plane are fixed in relation to the foci path, each prism has five degrees of freedom: four degrees of freedom for the rigid body \( \{ x_0, y, \phi, \beta \} \) and one degree of freedom for the aperture angle \( \alpha \). Here \( x_0 \) is the \( x \) coordinate of the center of the prism field surface and \( \{ \gamma, \phi, \beta \} \) are the rotation angles, respectively, around the axes \( \hat{x}, \hat{y} \) and \( \hat{z} \) of a coordinate system, which is fixed with respect to the center of the prism-field surface and is parallel to the axes \( x, y \) and \( z \), respectively.

Five constraints can be fixed by the system geometry. Three constraints are set by the requirement that the outcoming beam must impinge on the objective mirror exactly overlapping its source point. One constraint is set by the requirement that the optical path between a point source on the objective mirror and its image through the prism must be twice as long as the radius of curvature of the mirror.

The fifth constraint is expressed in the rotational
invariance around the prism axis that contains the apex of the aperture angle $\alpha$ (until the critical angle is exceeded).

It is noted that, for the optical design of the White cell, each time a prism is used as a retroreflector, the constraints that are due to the system geometry reduce the degree of freedom to zero and identify completely the set of variables \( \{x_0, \gamma, \phi, \beta, \alpha \} \). This means that we need a prism for each angular condition.

The optical configuration proposed by Ritz and Doussin uses standard prisms as retroreflectors for beams with different angular conditions. The prism \( D \) is crossed by four beams, each with different angular conditions, and the prism \( E \) is crossed by two beams, each with different angular conditions. Ritz and Doussin set the aperture angle and the alignment of each prism by the average values of the different constraints.

In the following chapters it will be shown that this setup produces in the best case a drastic reduction of light intensity and in the worst case a zero-light throughput. To optimize the light throughput we have designed a new White cell in which the prisms \( D \) and \( E \) are now replaced by two monolithic multiple-face (MMF) retroreflectors. These two retroreflectors are designed to satisfy the angular requirements for \( \alpha \) and \( \{\gamma, \phi, \beta\} \) for each light pass while the value of \( x_0 \) is set as an average value. The replacement of the two prisms with two MMF retroreflectors does not increase the alignment complexity.

To gain a better understanding of White cell behavior we developed a ray-tracing program. This program allows one to calculate the illumination of each optical part of the White cell for each given prism arrangement and to determine which one corresponds to the maximum-light throughput.

In contrast to laboratory experiments, simulations by computer programs also deliver direct information on the optical parts and steps that are difficult to check because of very weak illumination.

### 3. Retroreflector Coordinates

Once we establish the set of constraints, finding the coordinates \( \{x_0, \gamma, \phi, \beta, \alpha\} \) involves only a few geometric optics considerations (described in Appendix A).

We determine the value of \( \alpha \) by the evaluation of the roots of the transcendental function \( f \) in the range \( 0 \to 90 \) degrees:

\[
f = 1 - \frac{2 R n_I}{\Delta_{10}} + \frac{n_I}{\sin(n_I(90 - \alpha)\pi/180)} \left[ \frac{(h_p - \Delta_{10})}{\Delta_{10}} \{1 + \sin((90 - \alpha)\pi/180)\} \right],
\]

where \( R \) is the radius of curvature of the objective mirror, \( n_I \) is the refractive index of the retroreflector material, \( \Delta_{10} \) is the distance between the input and the output spots on the retroreflector field surface, and \( h_p \) is the length of the prism hypotenuse.

The function \( f \) has only one zero value over the \( 0 \to 90 \) degrees range.

Once \( \alpha \) is known,

\[
\beta = \arcsin\left\{2(p_p - m_p)\tan[n_I(90 - \alpha)\pi/180]/(\Delta_{10} C_{proj}) \right\} 180/\pi, \tag{2}
\]

\[
x_0 = R - \frac{\Delta_{10}}{2 \tan[n_I(90 - \alpha)\pi/180]} C_{proj} \cos\left(\frac{\beta}{180}\right), \tag{3}
\]

where, for the retroreflector \( D \), \( p_p \) is the \( y \) coordinate of the spot on the retroreflector field surface, \( m_p \) is the \( y \) coordinate of the center of the objective mirror \( A \), and \( z_p \) is the \( z \) coordinate of the center of the retroreflector field surface. For the retroreflector \( E \), \( p_p \) is the \( z \) coordinate of the spot on the retroreflector field surface, \( m_p \) is the \( z \) coordinate of the center of the objective mirror \( A \), and \( y_p \) is the \( y \) coordinate of the center of the retroreflector field surface.

The coefficient \( C_{proj} \) is defined as

\[
C_{proj} = (1 - (2z_f \tan[n_I(90 - \alpha)\pi/180]/\Delta_{10})^2)^{1/2}. \tag{4}
\]

Let us now consider a new coordinate system \( \{\tilde{x}, \tilde{y}, \tilde{z}\} \) obtained by rotation of \( \{x, y, z\} \) around the \( \tilde{z} \) axis by an angle \( \beta \). Once we define \( \{\gamma', \phi', \beta'\} \) as the rotation angles on the axes \( \tilde{x}, \tilde{y}, \tilde{z} \) around the \( \tilde{z} \) axis, we can set, for the retroreflector \( D \), \( \{\gamma', \phi', \beta'\} = \{\pi/2, 0\} \) and for the retroreflector \( E \), \( \{\gamma', \phi', \beta'\} = \{0, \pi/2\} \).

It is noted that in the \( \{\tilde{x}, \tilde{y}, \tilde{z}\} \) coordinate system the angles \( \phi' \) for retroreflector \( D \) and \( \gamma' \) for retroreflector \( E \) coincide with the retroreflector axis; that is, they are rotationally invariant until the critical angle is exceeded.

### 4. Multiple-Face Retroreflectors

As shown in the previous paragraph, \( \alpha \) is a function of \( \{\Delta_{10}, R, n_I\} \), \( \beta \) is a function of \( \{p_p, m_p, z_p, \Delta_{10}, n_I, \alpha\} \), and \( x_0 \) is a function of \( \{\Delta_{10}, R, n_p, z_p, \alpha, \beta\} \).

An exact solution for the retroreflector \( D \) requires it to behave like four different prisms. Considering that \( \{R, n_I, m_p, z_p\} \) are common to all four prisms and that the foci path reduce, respectively, \( \Delta_{10} \) and \( p_p \) to only two values each, we can write the matrix of the parameters of the four prisms as presented in Table 1. \( \Delta_1 \) and \( \Delta_2 \), respectively, are the distances between the input and the output on the retroreflector field surface for the spots \( \{10–11, 30–31\} \) and \( \{50–51, 70–71\} \).

<table>
<thead>
<tr>
<th>Spots</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( x_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10–11</td>
<td>( \alpha(\Delta_1) )</td>
<td>( \beta(\Delta_1, p_1, m_1, z_1) )</td>
<td>( x_0(\Delta_1, \alpha(\Delta_1), \beta(\Delta_1, p_1, m_1, z_1)) )</td>
</tr>
<tr>
<td>30–31</td>
<td>( \alpha(\Delta_2) )</td>
<td>( \beta(\Delta_2, p_2, m_2, z_2) )</td>
<td>( x_0(\Delta_2, \alpha(\Delta_2), \beta(\Delta_2, p_2, m_2, z_2)) )</td>
</tr>
<tr>
<td>50–51</td>
<td>( \alpha(\Delta) )</td>
<td>( \beta(\Delta, p_1, m_1, z_1) )</td>
<td>( x_0(\Delta, \alpha(\Delta), \beta(\Delta, p_1, m_1, z_1)) )</td>
</tr>
</tbody>
</table>

*See Fig. 2.*
shows that the roots of

design an easy-to-align system, we suggest the use of
degrees of freedom of the matrix of the parameters,

Fig. 3. Upper, $\beta$ as function of $\Delta_{10}$ for $p_1 = 68.5$ mm and $p_2 = 61.5$
mm; lower, $\beta$ as function of $p_1$, for $\Delta_1 = 31$ mm and for $\Delta_2 = 17$ mm.

70–71]. The $y$ coordinates are $p_1$ and $p_2$, on the
retroreflector field surface, for spots $\{10–11, 70–71\}$
and $\{30–31, 50–51\}$, respectively.

The numerical study, for a White cell with $R = 2000$ mm and $n_f = 1.46233$ (Suprasil at 500 nm),
shows that the roots of $f = 0$ for $\Delta_1 = 31$ mm and $\Delta_2 = 17$ mm are separated more than the expected production
error of $\pm 15$ arcsecond.

The retroreflector $D$ represented by Table 1 is feasible
by way of a set of eight independent planar
mirrors, or two monolithic prisms and four independent
planar mirrors. These solutions are complex and difficult to align.

The study of $\beta$ as a function of $\Delta_{10}$ and $p_p$, shows that
$\beta$ has a weak dependence on $\Delta_{10}$ and a stronger
dependence on $p_p$ (Fig. 3). On such a basis, and to
design an easy-to-align system, we suggest the use of
a MMF retroreflector, partially reducing the number
of degrees of freedom of the matrix of the parameters,
as presented in Table 2. Where $x_m = x_0[\Delta_m, \alpha(\Delta_m),\n\beta(\Delta_m, p_m, \alpha(\Delta_m))]$, $\Delta_m$ is the average between $\Delta_1$ and
$\Delta_2$ and $p_m$ is the average between $p_1$ and $p_2$.

It is important to note that this is the first guess for
the optical setup of the multiple-pass cell, and we can
only demonstrate the validity of such hypothesis a

_anteriori_ by propagating the light through the cell
with the ray-tracing program.

Starting from the parameters defined in Table 2,
we have designed a new MMF retroreflector $D$
shown in Fig. 4. The light that impinges on the
retroreflector surface in F1 is returned to the objective
mirror with an angle $\alpha_{D1} = \alpha(\Delta_1)$, whereas the light that impinges on F2 is returned to the objective
mirror with an angle $\alpha_{D2} = \alpha(\Delta_2)$. A dark gray line
and a pale gray line schematically represent the two
light paths inside the retroreflector.

The same approach is taken for $\beta$. Two retrorefectors
with the shape shown in the side view of Fig.
are glued along one of the side surfaces to form an
angle $\Delta_\beta = |\beta_{D1} - \beta_{D2}|$, where $\beta_{D1} = \beta(\Delta_m, p_1, \alpha(\Delta_m))$
and $\beta_{D2} = \beta(\Delta_m, p_2, \alpha(\Delta_m))$, as shown in the top view
of Fig. 4. Once one of the two pieces forming the
multiple-face retroreflector is aligned (matching, for
example angle $\beta_{D1}$); the other is also aligned, matching
angle $(\beta_{D1} - \Delta_\beta) = \beta_{D2}$.

Analogously, for the retroreflector $E$ an exact solution
requires two prisms. The parameters $\{\Delta_{10}, R, \n_{1_f}, m_p, z_i\}$ are common to both prisms, and the
ifferences between the two prisms are driven only by
the parameter $p_p$. The foci path requires that $p_p$
can take two values, $p_{E1}$ and $p_{E2}$, which are the $z$ coordinates
on the retroreflector field surface for spots $\{20–21\}$
and $\{60–61\}$ respectively.

To reduce the retroreflector $E$ to a monolithic one,
we have evaluated $x_0$ in $p_{Em}$, the average between $p_{E1}$
and $p_{E2}$. The set of parameters for the MMF retroreflector $E$ is reported in Table 3. Here, $x_{Em} = x_0[\Delta_E, \alpha(\Delta_E),\n\beta(\Delta_E, p_{Em}, \alpha(\Delta_E))]$, $\Delta_E$ is the distance between the input and the output on the retroreflector field surface for spots $\{20–21\}$.

The retroreflector $F$ is crossed only once, and it is
built up with a prism of angle $\alpha_F = \alpha(\Delta_p)$, whereas $\Delta_F$
is the distance between the input and the output on
the retroreflector field surface of spots $\{40–41\}$.

![Fig. 4. Side and top views of the multiple-face retroreflector D.](image)

<table>
<thead>
<tr>
<th>Table 2. Parameters of the MMF Retroreflector D</th>
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<tbody>
<tr>
<td>Spots$^a$</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>10–11</td>
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<td>30–31</td>
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<tr>
<td>70–71</td>
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$^a$See Fig. 2.

<table>
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<th>Table 3. Parameters of the MMF Retroreflector E</th>
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</tr>
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<tr>
<td>20–21</td>
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<tr>
<td>60–61</td>
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</tbody>
</table>

$^a$See Fig. 2.
5. Comparison between Theory and Ray Tracing

Having defined the previous geometry we have evaluated by computer simulation the performance of three different optical configurations: our MMF retroreflector configuration, the original optical configuration proposed by Ritz, and a computer optimization of the Ritz configuration carried out by us.

In the original optical configuration proposed by Ritz, the retroreflectors $E$ and $F$ are built up with two standard prisms, and the aperture angle $\alpha$ and the alignment of each prism are evaluated by the average values of $\Delta_{10}$ and $p_p$.

In our computer optimization of the Ritz configuration the retroreflectors $E$ and $F$ are built with two standard prisms, and the aperture angle $\alpha$ and the alignment of each prism are evaluated through a computer minimization of the light loss on the surface of the first objective mirror. We have minimized the rms of spot distance from the center of an objective-mirror surface. The minimization routine starts from the original optical configuration proposed by Ritz.

We have simulated a 2-m White cell of $f/40$ aperture. The cell consists of a square ($100 \times 100$ mm) spherical field mirror with a 2000-mm radius of curvature, two 50-mm-diameter spherical objective mirrors with a 2000-mm radius of curvature, and three prisms. For prisms $D, E,$ and $F$ the surfaces opposed to the apex have sizes of $24 \times 48$ mm, $14 \times 30$ mm and $7 \times 14$ mm, respectively.

In all optical configurations the center of curvature of the field mirror lies halfway between the two objective-mirror surfaces. The center of curvature of the objective mirror $B$ lies on the center of the field-mirror surface, whereas that of $A$ lies on the field-mirror surface horizontally shifted from the center by an amount $m_d$. This displacement drives the number of light passes in the cell. For the simulation described here we have taken a value of $m_d$ corresponding to 144 passes, as shown in Fig. 2.

To investigate the performance of the optical design we have propagated three different sources: a single ray, a bundle of parallel rays, and a bundle of cones of rays.

First, we have propagated a single-ray source. The ray starts from the input and is aligned to impinge on the surface of the objective mirror $A$ in the center. Figure 5 shows the results at the front view of the two objective mirrors for the three optical configurations in the single-ray source case. Each spot on the surface of an objective mirror represents the five-times cross resonance inside the basic White cell. For the basic White cell we intend the optical subsystem to be composed only of spherical mirrors.

Figure 5 demonstrates how strongly the retroreflector configuration influences the spread of the ray spots on the objective-mirror surfaces and consequently the light efficiency of the multiple-pass cell.

In the original Ritz retroreflector configuration (Fig. 5, middle) some spots fall outside of the actual objective mirrors, and consequently the light transmission is zero.

To use fully the available diameter of the objective mirror, let us consider an ideal point source of $f/40$ aperture. In this case the computer optimization of the Ritz configuration carried out by us (Fig. 5, lower) still has high light loss, whereas the MMF configuration (Fig. 5, upper) has negligible light loss. Furthermore, the ray-tracing program shows that on the field surface, i.e., mirror $B$ and retroreflectors, the focal path is not appreciably influenced by the retroreflector configuration.

Since only the MMF configuration has an acceptable behavior, we consider neither the original Ritz configuration nor our computer optimization of it.

This approach can be implemented by simulating the actual diameter of the light input. Thus we consider a source composed of a bundle of five parallel rays. One ray is set in the center of the input spot, and four rays are set on the corners of a square centered on the input spot with its diagonal equal to the input-spot diameter. The bundle source starts from the input and is aligned to impinge upon the surface of the objective mirror $A$ in the center. Figure 6 shows the front view of the objective mirror $A$, for the MMF retroreflector configuration in the five-parallel-ray source case. Each picture of the se-
quence represents 18 passes inside the basic White cell.

Figure 6 shows that the source diverges more and more after each prism crossing. This behavior is due to the breakdown of the symmetry of the basic White cell introduced by the retroreflectors.

For the basic White cell the White rule\(^1\) says that the object-point position and the image-point position of successive images, near the center of curvature of a spherical mirror, always lie on a straight line whose midpoint falls in the center of curvature.

After the five-ray source has illuminated the objective mirror \(A\), we focus the light halfway between the objective mirror and the field mirror, and the beam diverges as it impinges upon the field mirror. The field mirror focuses the light on the objective mirror \(B\) to form approximately a spot that is specular to its source on the objective mirror \(A\). Pass after pass, the basic White cell maintains the size of the spots on the objective-mirror surfaces.

After 18 passes the light leaves the basic White cell, and the beam diverges as it impinges upon the retroreflector. The retroreflector basically acts as a planar mirror that doubles the light path and leaves the beam divergence unchanged. Consequently the spot that impinges back on the objective mirror \(A\) is enlarged.

The light then makes 18 passes through the basic White cell, maintaining, unchanged, the new light divergence impinging on the objective mirror. When the light crosses the prisms the spot impinging back on objective mirror \(A\) is further enlarged. The same happens for each prism crossing. It is important to note that this process is independent of the particular retroreflector configuration.

Using the paraxial approximation for a spherical mirror, we compute the size of the spot impinging back on objective mirror \(A\) as

\[
H = (1 + 2n)h,
\]

where \(n\) is the number of crossed prisms and \(h\) is the size of the input image.

As a consequence of the ray-tracing results obtained for both the single-ray and the five-ray source, 80-mm-diameter objective mirrors (instead of the original 50 mm) are necessary to obtain a 100% geometric light transmission for the MMF retroreflector White cell of \(f/40\) aperture.

Let us now consider an MMF retroreflector White cell with 80-mm-diameter objective mirrors. To test our new optical design further, we have propagated a source composed of five cones of rays. Each cone of rays has a \(f/40\) aperture and consists of 20 single rays. The axes of the cones are parallel and are disposed with one in the center of the input spot and four on the corners of a square, centered on the input spot, the diagonal of which is equal to the input-spot diameter. The parallel-cones source starts from the input and is aligned so that the cone in the center of the input spot impinges upon the surface of the objective mirror \(A\) in the center.

Figure 7 shows the front view of the objective mirror \(A\) for our MMF retroreflector configuration for the five \(f/40\) parallel-cones source. Each picture of the sequence represents 18 passes inside the basic White cell.
cell. Figure 7 shows that the MMF retroreflector configuration has 100% geometric light transmission.

Figure 8 shows the foci path on the field surface, i.e., field mirror B and retroreflectors, for the MMF retroreflector configuration in the five f/40 parallel-cones case.

We have compared the relative light efficiency of the three retroreflector configurations discussed in this paper by evaluating numerically the loss that is due to illumination mismatching on the objective mirrors.

We carried out the calculation by considering 90 source points of f/40 aperture that have unit-step energy shape. The source points are uniformly distributed on the input spot of 15-mm diameter.

For our setup, the numerical results normalized to the light transmission of the MMF retroreflector configuration show a light transmission of 82% for our computer optimization of the Ritz configuration and 0% for the original Ritz configuration.

6. Feasibility

Among the parameters that define a retroreflector configuration, the production tolerance influences the accuracy of the angles $\alpha$ and $\beta$ of the retroreflectors $D$ and $E$ and the angle $\alpha$ of the retroreflector $F$.

The MMF retroreflector configuration defines eight manufactured angles, $\{\alpha_{D1}, \alpha_{D2}, \beta_{D1}, \beta_{D2}\}$ for retroreflector $D$, $\{\alpha_E, \beta_{E1}, \beta_{E2}\}$ for retroreflector $E$, and $\{\alpha_F\}$ for retroreflector $F$. We have evaluated how the expected production errors of the retroreflector angles influence the alignment of the MMF retroreflector configuration.

Medium-to-high-quality optics are manufactured with an angle tolerance of $\pm 15$ arcsec. For each angle we consider the exact value of the MMF retroreflector configuration and a variation of $\pm 15$ arcsec. The number of possible multiple-pass configurations that should be examined in varying the prism angles increases to $3^8$.

As stated in the previous chapter the MMF retroreflector configuration is characterized by the angles $\Delta \beta_D = |\beta_{D1} - \beta_{D2}|$ and $\Delta \beta_E = |\beta_{E1} - \beta_{E2}|$ of the retrorefectors $D$ and $E$, respectively. To reduce computing time we evaluated only the worst case by taking only the variation with opposite sign for $\beta_{D1}$ and $\beta_{D2}$ and for $\beta_{E1}$ and $\beta_{E2}$, respectively. The number of parameters is reduced to six and the number of possible multiple-pass configurations that should be examined in varying the prism angles decreases to $3^6$.

In the single-ray source case we consider the maximum and rms of spot distance from the center of an objective-mirror surface, $d_{\text{Max}}$ and $d_{\text{RMS}}$, respectively. For each set of parameters defining a different configuration, we ran the ray-tracing program and calculated $d_{\text{Max}}$ and $d_{\text{RMS}}$. Each set of parameters is labeled with an integer corresponding to the iteration process applied to the ray tracing. The scatterplot of the values of $\Delta d_{\text{Max}} = d_{\text{Max}} - d_{\text{Max,0}}$, where $d_{\text{Max,0}}$ is evaluated in the exact MMF retroreflector configuration, is shown in Fig. 9.

Figure 9 shows that in the worst case $\Delta d_{\text{Max}}$ is less than 3.5 mm. The expected production errors can be compensated through use of slightly larger objective mirrors to maintain 100% geometric light transmission.

The multiple-pass cell described here has been designed for differential optical absorption spectroscopy measurements. This technique requires the simultaneous measurement over a spectral range of the order of several tenths of a nanometer. The use of quartz retroreflectors produces a chromatic aberration. We have studied the dependence of the focus position along the x axis versus wavelength.

In the range of 200–2000 nm, the refractive index of suprasil shows a stronger dependence on wavelength in the UV range. In considering only the worst chromatic aberration we show the results of the simulation in the 250–350 nm range.

Let us consider an f/40 aperture source composed of a cone of 20 rays. The parameters of the retroreflectors are evaluated with use of the refractive index of suprasil quartz at 300 nm (1.48779 at 300 nm).
The results of the ray-tracing simulation are shown, over the range 250–350 nm, in Fig. 10. The value $d_{\text{focus}}$ is the difference between the focus position along the $x$ axis at wavelength $\lambda$ and the value at $\lambda = 300$ nm.

Figure 10 shows a variation of the order of 2.6 mm over the 100-nm spectral range. In our laboratory the output of the multiple-pass cell is collected from a spectrometer with an optical aperture of $f/7$. A reflective optical system is used to couple the $f/40$ aperture of the multiple-pass cell with the $f/7$ aperture of the spectrometer. The change of aperture compresses the focus variation by a factor of 7/40.

For measurements requiring wide spectral ranges the result of Fig. 10 suggests replacing the refractive retroreflectors with reflective elements. For the reflective monolithic retroreflector $D$ the surface shown in Fig. 4 was obtained with use of a concave monolithic system of eight planar mirrors. Analogously the same is true for retroreflectors E and F.

7. Conclusions

The theory of multiple-pass cells based on the use of retroreflectors has been presented. The theory shows that for a White multiple-pass cell, using prisms, the optical aperture of the cell cannot be calculated as simply the ratio between the diameter of the first objective mirror and the base path length.

If the light beam crosses the same prism more than once, the theory shows that the back image, which impinges the objective mirror, can no longer exactly overlap its source. Furthermore, the use of any kind of retroreflector breaks the White rule. As a consequence, the beam that impinges back on the objective mirror coming from a retroreflector is enlarged. The enlargement is proportional to the number of times that the light has crossed retroreflectors. The derived mathematical equations allow the design of a very long path cell with zero geometric loss.

A new MMF retroreflector has been proposed, and its use in the design of a multiple-pass cell has been tested with a ray-tracing simulation program. The feasibility of the system has also been studied.

8. Appendix A

Let us consider the plane that contains the center of the objective-mirror surface and the input and output points on the prism-field surface. Here the prism-field surface is the prism surface containing the hypotenuse. Once the prism is aligned correctly, this plane is perpendicular to the prism apex. Figure 11 shows a prism view in this plane.

Here $h_p$ is the length of the prism hypotenuse, $\Delta IO$ is the distance between the input and the output points on the prism surface, and $\phi$ is the angle with the apex at the center of the objective-mirror surface and is described between the outgoing and incoming beam.

We can express the constraint that the outgoing beam must impinge on the objective mirror exactly overlapping its source point as

$$\omega_1 + \omega_2 + \phi/n_I = 180,$$

where $n_I$ is the refractive index of the prism.

Combining the light reflection law (2 $\delta_1 + \omega_1 = 180$ and 2 $\delta_2 + \omega_2 = 180$), the sum of internal angles of the triangle ($\delta_1 + \delta_2 + \alpha = 180$), and the previous equations it follows that

$$\alpha = 90 - \frac{\phi}{2n_I}. \quad (A1)$$

In the case where the center of the objective-mirror surface lies on the prism symmetry axis, $b$ is parallel to the prism hypotenuse, and therefore $\psi = \phi$; the triangle described by $a$ and $(h_p - \Delta IO)/2$ is isosceles and therefore $a = (h_p - \Delta IO)/2$. 

--

Fig. 10. Focus position along the $x$ axis versus the wavelength in the MMF retroreflector configuration. The value $d_{\text{focus}}$ is the difference between the focus position along the $x$ axis at wavelength $\lambda$ and the value at $\lambda = 300$ nm.

Fig. 11. View of the prism in the plane perpendicular to its apex that contains the input and output points on the prism-field surface.
The length of $b$ is therefore

\[
b = \frac{\Delta_{10}}{2} + a \sin(\phi_0/\pi/360) = \frac{\Delta_{10}}{2} + \frac{h_p - \Delta_{10}}{2} \sin[\phi/\pi/(n_t 360)],
\]

where $\phi_0$ is the angle that the beam forms with the perpendicular of the field prism surface inside the prism, $\phi_0 = \phi/n_I$.

The distance $l$ between the center of the objective-mirror surface and the input point on the prism-field surface is

\[
l = \frac{\Delta_{10}/2}{\sin(\phi/\pi/360)}.
\]

The constraint that the optical path is twice the objective-mirror radius of curvature $R$ can be written as

\[
a + b = \frac{n_t}{\sin(\phi/\pi/360)} + l = R.
\]

Combining the previous equations of $a$, $b$, and $l$ we obtain

\[
R = \frac{1}{n_t} \left( \frac{\Delta_{10}}{2} + \frac{h_p - \Delta_{10}}{2} \{1 + \sin[\phi/\pi/(n_t 360)]\} \right) + \frac{\Delta_{10}/2}{\sin(\phi/\pi/360)},
\]

which, with Eq. (A1), gives

\[
1 - \frac{2Rn_I}{\Delta_{10}} + \frac{h_p - \Delta_{10}}{\Delta_{10}} \{1 + \sin[(90 - \alpha)\pi/180]\} + \frac{n_I}{\sin[n_I(90 - \alpha)\pi/180]} = 0. \tag{A2}
\]

The value of $\alpha$ is determined by evaluation of the root of the previous equation in the range $0–90$ degrees.

Let us now consider the center of the prism-field surface $P = \{x_0,p_p,z_f\}$ and the center of the objective-mirror surface $P_0 = \{R, m_p, 0\}$. Figure 12 shows $P$ and $P_0$ in a Cartesian coordinate system. Since the coordinates $p_p$, $z_f$, and $m_p$ are fixed, we can evaluate the angle $\beta$ and the coordinate $x_0$. Here $\beta$ is the rotation angle of the prism around the axis $z$ for which the light beam impinges perpendicularly on the prism surface in the $x$-$y$ plane.

Geometric considerations lead to the following equations:

\[
d' = d \cos(\beta\pi/180),
\]

\[
z_f = d \sin(\beta\pi/180),
\]

\[
p_p - m_p = d' \sin(\beta\pi/180),
\]

where $d$ is the distance between the center of the prism-field surface and the center of the objective-mirror surface:

\[
d = \frac{\Delta_{10}/2}{\tan[n_I(90 - \alpha)\pi/180]}. \tag{A3}
\]

If we combine the previous equations of $d'$ and $z_f$, it follows that

\[
d' = d(1 - (z_f/d)^2)^{1/2} = d C_{proj},
\]

where $C_{proj}$, the coefficient of projection of $d$ on the $x$-$y$ plane, is defined as

\[
C_{proj} = [1 - (z_f/d)^2]^{1/2} = (1 - (2z_f \tan[n_I(90 - \alpha)\pi/180]/\Delta_{10}^2)^{1/2}.
\]

The rotation angle $\beta$ of the prism is therefore

\[
\beta = \arcsin((p_p - m_p)/(d(1 - (z_f/d)^2)^{1/2})) \tan(90 - \alpha)/n_I \pi/180 (\Delta_{10}/C_{proj})^{1/2} \tag{A3}
\]

Along the $x$ axis, the constraint that the optical path is twice the objective-mirror radius of curvature is reduced to $x_0 = R - d_0$, where $d_0 = d' \cos(\beta\pi/180)$ is the projection of $d'$ along the $x$ axis.

The coordinate $x_0$ of the prism is therefore

\[
x_0 = R - \frac{\Delta_{10}/2}{\tan(n_I(90 - \alpha)\pi/180)} C_{proj} \cos(\beta\pi/180). \tag{A4}
\]

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References